Binary Trees

Outline

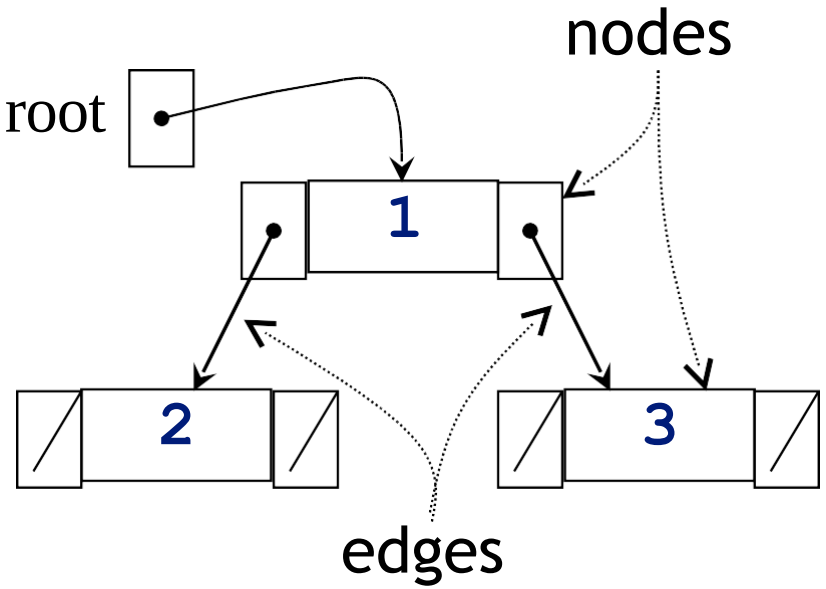
- Why Trees?
- Trees
- Tree Terminology
- Binary Trees
- One Recursive Tree Algorithm

Storing Many Objects

- We have examined 2 major ways to store data in the main memory of the computer
  - arrays
    - use subscripts to immediately access elements
    - fast access, but in the old days would consume more memory that you would like it to (not true anymore)
  - linked structures
    - each node refers to the next in the collection. To get to one you often traverse sequentially
    - maybe slower, but maybe manages memory better

Another Linked Structure

- We now turn our attention to another major way of storing data: the Tree
  - One implementation of a *binary* tree has nodes with a left and right link field

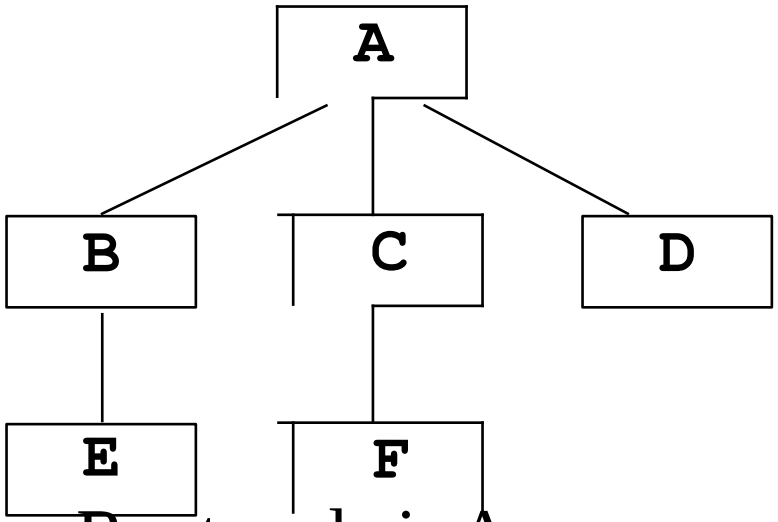


First Some Definitions

- A tree has a set of nodes and directed edges that connect them
  - a directed edge connects a parent to its children
- Tree properties
  - one node is distinguished as the root
  - every node (except the root) is connected by an edge from exactly one other node
  - A unique path traverses from the root to each node

General Trees

- Trees store data in a hierarchical manner



Trees have layers of nodes, where some are higher, others are lower.

  - Root node is A
  - A's children are B, C, and D
  - E, F, and D are leaves
  - Length of path from A to E is 2 edges

Some tree terminology

Node An element in the tree *references data and other nodes*

Root The node at the top *It is upside down!*

Parent The node directly above another node (except root)

Child The node(s) below a given node

Size The number of descendants plus one for the node itself

Leaves Nodes with no children

Levels The root A is at level 0, E and F are at level 2

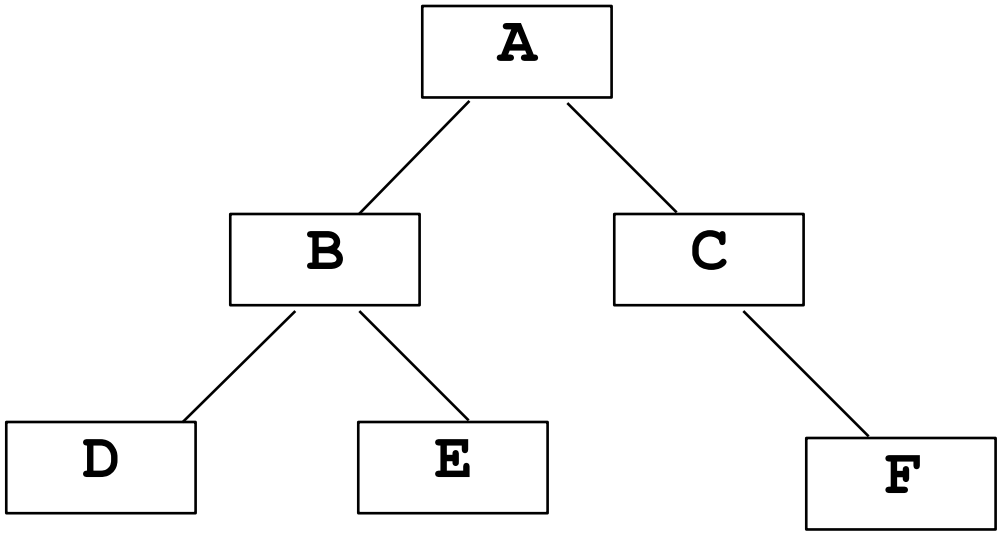
Applications of trees

- File Systems
  - Hierarchical files systems include Unix and DOS
  - In DOS, each \ represents an edge (In Unix, it's /)
  - Each directory is a file with a list of all its children
- Store large volumes of data
  - data can be quickly inserted, removed, and found
- Data structure used in a variety of situations
  - implement data vase management systems
  - compilers: expression tree, symbol tree
- The Computer Science Department's new logo

  okay we're working on a new logo

Binary Trees

- N-ary tree has n children max from each node
- A binary tree is a tree where all nodes have zero, one or two children *we'll study these binary trees only*
- Each node is a leaf, has a right child, has a left child, or has both a left and right child



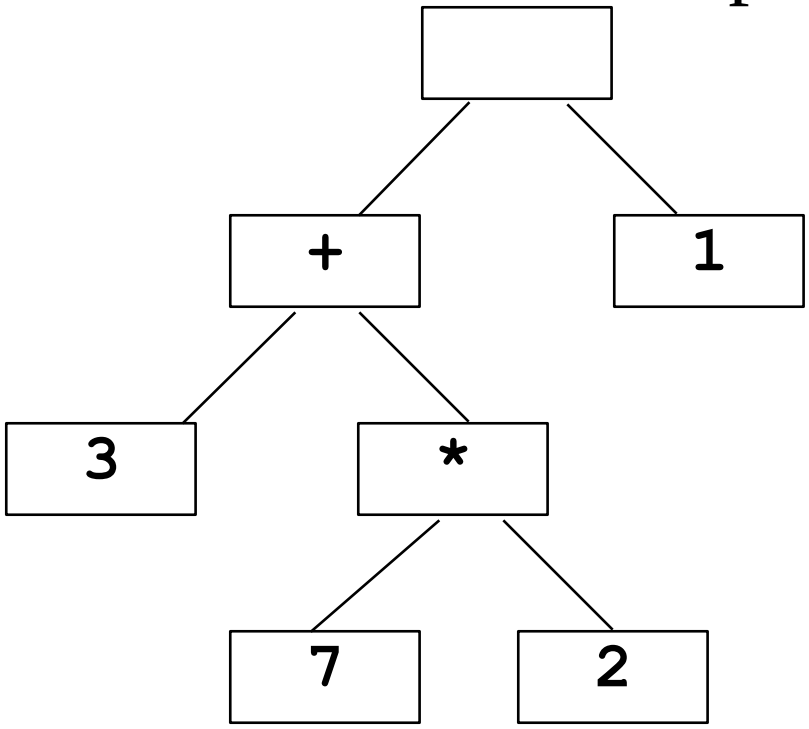
Binary Tree Defined

- A binary tree is either:
  - an empty tree
  - consists of a node, called a root, and zero, one, or two children (left and right), each of which are themselves binary trees
- This recursive definition uses the term "empty tree" as the base case
- Every non-empty node has two children, either of which may be empty
  - On previous slide, C's left child is an empty tree.

Application: Expression Trees

- Binary trees can represent arithmetic expressions

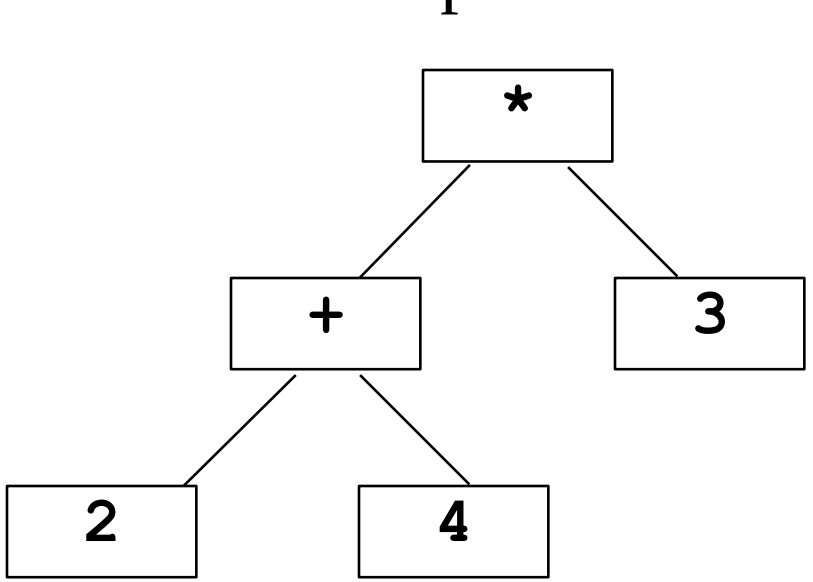
- An infix expression will have a parent operator and two children operands:

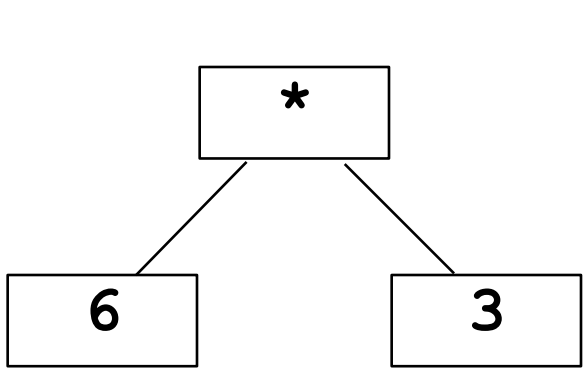


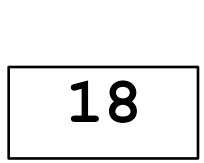
The expression:
`((3+(7*2))-1)`
Each parenthesized expression becomes a tree. Each operand is a leaf, each operator is an internal node

Evaluating the Expression tree

- To evaluate the expression tree:
  - Take any two leaves
  - Apply the parent's operator to them
  - Replace that operator with the value of the subexpression.

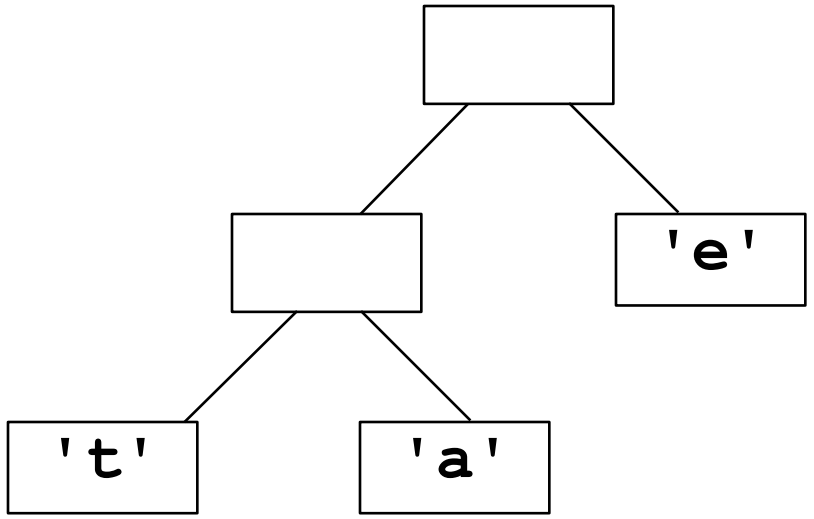






Huffman Coding Tree

- Binary trees in a famous file compression algorithm *Huffman Coding Tree*
- Each character is stored in a leaf
- The code is found by following the path
  - 0 go left, 1 go right
  - a is 01
  - e is 1
  - what is t?
  - What is `0100100101`**?**



An inner class will be used to store one node of a binary tree

```
private class BinaryTreeNode
{
  // instance variables
  private Object data;
  private BinaryTreeNode left;
  private BinaryTreeNode right;

  BinaryTreeNode()// This would be given to us
  {
    data = null;
    left = null;
    right = null;
  }
```

A Third Constructor

```
  BinaryTreeNode(Object theData)
  { // Used most often
    data = theData;
    left = null;
    right = null;
  }

  BinaryTreeNode(Object theData,
                 BinaryTreeNode leftLink,
                 BinaryTreeNode rightLink)
  {
    data  = theData;
    left  = leftLink;
    right = rightLink;
  }
}  // end class BinaryTreeNode
```

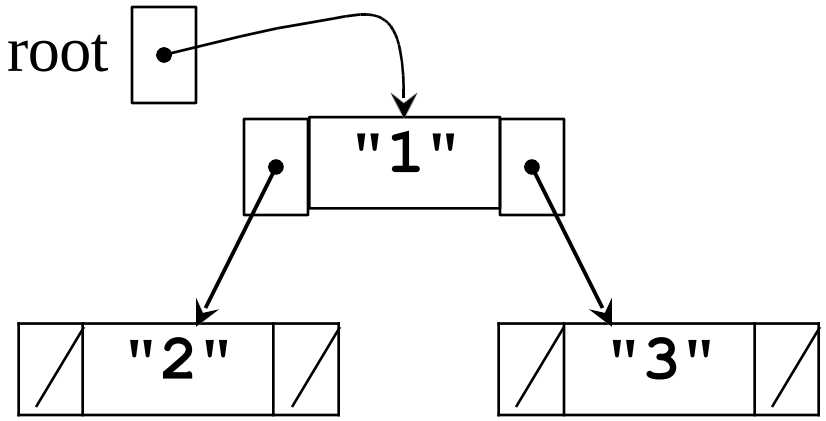
class BinaryTreeNode

- Each **`BinaryTreeNode`** object has
  - a reference to an object *object so we can store anything*
  - a link to the left subtree *which could be an empty tree*
  - a link to the right subtree *which could be an empty tree*
- 3 Constructors
  - two set some data fields to **`null`** (**`left`** and **`right`**)
- The data fields are private
  - Like **`LinkNode`**, methods in the enclosing class can reference private instance variables of the inner class

Build a Tree with Three Nodes

- Hard code a tree referenced by root
  - We do not yet have a nice way to insert new nodes
  - This demonstrates linked **`BinaryTreeNode`** objects

```
BinaryTreeNode root = new BinaryTreeNode("1");
root.left = new BinaryTreeNode("2");
root.right = new BinaryTreeNode("3");
```



Recursion and Trees

- Trees are defined recursively
  - and tree algorithms are implemented recursively
- For example, size (all descendants plus 1)

```
// call a size after building a tree
System.out.println(size(root));

public int size(BinaryTreeNode t)
{ // return size of tree rooted at t
```

What is the base case? (think simplest possibility)

With trees, the recursive case often makes a recursive call on the left subtree and another on the right subtree

```
}
```

Active Learning

1) Draw this tree

```
BinaryTreeNode aTree = new BinaryTreeNode("1");
aTree.left = new BinaryTreeNode("2");
aTree.right = new BinaryTreeNode("3");
aTree.right.right = new BinaryTreeNode("4");
aTree.right.left = new BinaryTreeNode("5");
aTree.left.right = new BinaryTreeNode("6");
aTree.left.left = new BinaryTreeNode("7");
aTree.left.left.left = new BinaryTreeNode("8");
```

2) Then write output generated

```
System.out.println(size(aTree));
// Of course you could keep track of
// size as another instance variable
```